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Determination of GPS Coordinate Transformation Parameters of Geodetic Data between Reference Datums-A Case Study of Ghana Geodetic Reference Network

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Abstract

The issues of datum transformation of GPS coordinates from the War Office to the WGS84 coordinate systems are a real, persistent and serious problem in Ghana that requires pragmatic measures for solutions. The Ghana national mapping coordinates data are based on a non-geocentric coordinate system known as the War Office of 1926, which like many others around the world was realized through conventional methods of surveying. Therefore, GPS based WGS84 coordinates data are not compatible with the War Office. This study aims at investigating and comparing the similarity (abridged Molodensky, Bursa-Wolf, Veis and Molodensky-Badekas) and position dependent (3D projective and 12-parameter linear affine) transformation models for the study area by determining their associated transformation parameters so that users from all related disciplines can utilize them without any problem. To achieve the aims of this research, nineteen common points between the War Office and WGS84 system were used to estimate the transformation parameters. In order to ascertain the precision of the derived transformation parameters, the reference standard deviation, reference adjustment variance and the individual standard deviations of the parameters determined were used. Residual analysis using maximum negative, maximum positive, mean error and standard error was then conducted to evaluate the performance of the transformation models. The test of normality of residuals, deterministic model, correlation coefficient r , t-test on the derived transformation parameters, t-test on the correlation coefficient, Root Mean Square error and Root Mean square error distribution were used to evaluate and check the adequacy of the transformation models. Tests were carried out on nineteen points with coordinates known in both WGS84 and War Office systems. The results indicated that the Bursa-Wolf, Molodensky-Badekas and Veis models parameters transform WGS84 coordinates to War Office coordinates accurate to ± 1 m and the abridged Molodensky parameters produce War Office coordinates accurate to ± 1.9 m. In the case of the position dependent methods, the 3D projective model is in the order of approximately ± 1.7 m and ± 0.8 m for the 12 parameter linear affine model. Based on the results it was found that both similarity and position dependent methods generate results consistent with their accuracies that have been quoted in scientific literature.

Keywords: Global Position Systems, Coordinate Systems, Transformation models.

Introduction

In the broad spectrum of activities covered by geodesy, one of the primary tasks is the establishment of a well-defined coordinate system and datum for accurate positioning on the Earth surface. These coordinate systems or datums, may be of a local or regional nature or even of global extent, have a variety of uses in the realms of both scientific and applied geodesy. For example, recently, there is an increasing interest in positioning techniques based on Global Navigation Satellite Systems (GNSS) such as Global Positioning System (GPS) in Ghana.

GNSS furnishes the principal technology for geomatics and geodetic activities in Ghana. It is well acknowledged that the use of GPS as a cost-effective tool for the majority of future geodetic surveys is on the ascendancy in Ghana (Dadzie *et al.*,

2008) due to its numerous advantages over classical methods of surveying. However, the use of GPS as a tool for geodetic positioning in Ghana has some limitations. Thus, no generally accepted standards exist for its use in Ghana due to inconsistencies in the transformation parameters derived among researchers. As a result, users adopt different methods in the post-processing of the GPS observational raw data and these generate a lot of uncertainty and inconsistency in the use of the GPS data from different sources (Dzidefo, 2011). Therefore, the rapid growth of utilizing GPS technology in Ghana necessitates the derivation of transformation parameters for transforming

observational data from War Office to WGS84 datum.

The issue of transforming GPS-based coordinates to the national coordinates system and vice versa has become a crucial concern for geodetic and geomatics applications (Dawod *et al.*, 2011). For example, geospatial and non-geospatial professionals in their day-to-day activities are challenged with the task of integrating geodetic information based on two different incompatible geodetic datums. The problem of having different datums was not recognized as a major problem in the past because most Geographic Information Systems (GIS) applications have been applied on small or medium scales where errors due to different datums of up to 100-500 m do not show up (LUPMIS, 2012). The complexity of this incompatibility between different mapping systems or datums is of general concern to those involved in the collection of natural resources data where projections and conversion between datums are required. Graham *et al.* (2000) emphasized that “the Ghana National Grid (GNG) is an example of a mapping system that is not defined in image processing and Geographic Information System (GIS) software and for which the transformation parameters are not readily available in the literature. Therefore, integrating GNG topographic map data within a GIS with data derived from other sources can be problematic”. Therefore, there is much need to determine transformation parameters that will mathematically convert observed GPS based data to the national geodetic reference system in order to avoid discrepancies caused by the change of geodetic datum data from one geodetic datum to another. For instance, the Ghana Geodetic Datum has the War Office ellipsoid adopted as the reference ellipsoid to meet the requirement for mapping and engineering projects in Ghana. Consequently, using GPS as a tool for geodetic positioning in Ghana requires that the datum transformation from WGS84 to the Ghana geodetic datum (War Office) have to be defined. Efforts have been made by researchers (for example, Poku-Gyamfi and Schueler, 2008; Ayer and Fosu, 2008; Ayer and Tienhah, 2008; Ayer, 2008; Dzidefo, 2011) in Ghana and foreign agencies to determine an appropriate procedure and transformation parameters for the Ghana Geodetic Reference Network called the Golden Triangle. No conclusion has been reached as to which of them is the preferred approach and no doubt this is an area where discussions will continue for some time to come. As a result, an optimal set of transformation parameters between the Accra and WGS84 datum does not exist, this makes it difficult for Ghana to utilize the massive potential of GPS

services. In order to establish compatibility in data obtained from GPS measurements correctly and effectively in Ghana, it is necessary to determine appropriate transformation parameters that relate the coordinates in the War Office system to the WGS84 system using relevant transformation models.

Several transformation models have been put forth to compute transformation parameters for geodetic reference network of countries (Thomson, 1994; Hofmann-Wellenhof *et al.*, 1997; Constantin-Octavian, 2006; Ayer and Tienhah, 2008). The most commonly used among these models are the conformal similarity methods such as Helmert Similarity Transformation, Bursa-Wolf and Molodensky-Badekas because of their ease in application and resulting accuracy (Kutoglu *et al.*, 2002). In this study, four similarity coordinate transformation methods namely; Bursa-Wolf Model, Molodensky-Badekas Model, Veis Model and abridged Molodensky model were studied. In addition, two position-dependent models namely; three-dimensional polynomial projective transformation and 12-parameter linear affine transformation were also investigated. The position dependent techniques applied in this study were chosen to experiment on its application to global and local geodetic datums transformation and how much they differ from the seven-parameter similarity methods. This idea was motivated by the fact that their applications are mostly used in the field of photogrammetry and remote sensing and limited literature is available on its application on global and local datums especially in developing countries like Ghana where geodesy has not yet reached the advance stage. It is worth mentioning that these position dependent methods are widely used within local systems but rarely applied for global and local datums transformation parameter determination.

Transformation parameters and tests results obtained from the similarity and position dependent models for transforming curvilinear coordinates from Ghana geodetic datum (War Office) to WGS84 datum and vice versa were compared. The study is therefore limited to; the determination of transformation parameters within Ghana Geodetic Reference Network (GGRN) between the War Office datum and the WGS84 datum; estimation of the accuracy measures of the models; normality test of the residuals; a hypothesis test on the derived transformation parameters; a comparative study of the above mentioned models and finally evaluating the performance of the models using existing coordinates information within the GGRN.

Materials and Methods

Study area

Ghana is located in West Africa, bordered by Cote D'Ivoire to the West, Togo to the East, Burkina Faso to the North and the Gulf of Guinea to the South. The country spans an area of 239,460 sq. km with the land mass generally consisting of low plains with a dissected plateau in the south-central area and scattered areas of high relief (Baabereyir, 2009). Lying just above the equator, Ghana has a tropical climate with mean annual temperatures ranging between 26°C and 29°C but temperatures are generally higher in the North than in the South (Baabereyir, 2009). Ghana lies between latitudes 4⁰ and 12⁰ N and longitude 4⁰ E and 2⁰ W (Anon, 2013). The country is divided into ten administrative regions as shown in figure 1 below.



Figure 1: Map of Ghana showing the study area (Anon, 2013)

This study covers five out of the ten administrative regions in Ghana. That is; Ashanti, Greater Accra, Western, Central and Eastern (figure 1). These regions form the first phase of the new established geodetic reference network referred as the Golden triangle. The Golden Triangle of Ghana has the three largest cities and covers a little over a third of the country with 58% of the total population (Poku-Gyamfi and Schueler, 2008). These regions have almost all the natural resources such as gold, bauxite, manganese, oil, timber found in the country and thus are of high economic importance. Three permanently operating reference stations have been established at the vertices of this triangle with eighteen-second order reference stations spatially well distributed in this area in question.

Materials

Primary data of geodetic coordinates for both WGS84 and War Office ellipsoid were collected from the Ghana Survey Department with secondary data obtained from research publications to determine the transformation parameters. Computer programming codes in Matlab (2012b) was written to handle the parameter estimation for the various transformation models. Plotted control points, data structures, descriptive and summary statistics for the entire study were also produced using Matlab (2012b), Microsoft Excel 2007, SPSS (Version 20) and Surpac (Version 6.1.2).

Methods

Curvilinear geodetic coordinates of common points in both the WGS84 and War Office system were first converted into rectangular cartesian coordinate (X, Y, Z). To accomplish this task, the following relationships were applied;

$$X = \frac{a \cos \phi \cos \lambda}{[1 - f(2 - f) \sin^2 \phi]^{1/2}} + h \cos \phi \cos \lambda$$

$$Y = \frac{a \cos \phi \sin \lambda}{[1 - f(2 - f) \sin^2 \phi]^{1/2}} + h \cos \phi \sin \lambda \quad (1)$$

$$Z = \frac{a(1 - f)^2 \sin \phi}{[1 - f(2 - f) \sin^2 \phi]^{1/2}} + h \sin \phi$$

Where;

h = ellipsoidal height

a = semi major axis of the reference ellipsoid

f = flattening which measures the differences between the two axes of an ellipsoid.

(ϕ, λ) = geodetic latitude and geodetic longitude

The ellipsoidal parameters used in equation (1) for WGS84 are; semi-major axis value of 6378137.0 m and a flattening of 1/298. The War Office on the other hand has a semi-major value of 6378299.99899 m and a flattening of 1/296.

Equation (1) was applied directly to the WGS84 curvilinear geodetic coordinates (ϕ, λ, h) to obtain its related rectangular cartesian coordinates (X, Y, Z)_{WGS84}. However, equation (1) cannot be readily applied directly in deriving the rectangular cartesian coordinate for the War Office ellipsoidal system. This is because data in the War Office ellipsoid expressed as (ϕ, λ, H) contains only orthometric height. Hence, the applicability of equation (1) is limited in this scenario. In view of this development, the iterative abridged Molodensky

transformation model equation (Deakin, 2004) was first applied to the War Office data (ϕ, λ, H) to determine the approximate (change) difference in ellipsoidal height, Δh . This was then used to estimate h for the War Office using $h_{war} = h_{WGS84} - \Delta h$. Having derived the war office ellipsoidal height, equation (1) was then applied to calculate the rectangular cartesian coordinates for the War Office datum. Hence, there exist common points of all coordinates in both systems at this stage. The above mentioned transformation models used in the study were executed on the cartesian coordinates of common points in both systems to determine a sets of transformation parameters. These parameters were estimated via parametric least square adjustment technique expressed mathematically as $V + BX = f$ (2)

Where:

V = the vector of residuals

B = the design matrix

X = the vector of parameter estimates and

f = the vector of observations.

In all, nineteen control stations (figure 2) tied to both reference systems within the geodetic reference network were used for the parameter estimation. Figure 2 below has a black background because of the Surpac software used to do the plotting.

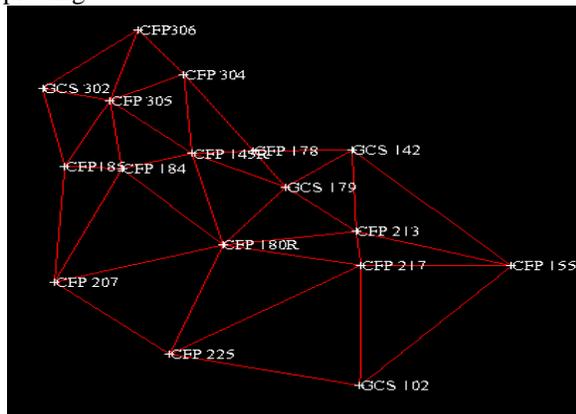


Table 1: Summary of the derived parameters results

Parameter	Conformal Similarity Transformation Models				UNITS
	Abridged Molodensky	Bursa-Wolf	Veis	Molodensky-Badekas	
ΔX	-196.63455±0.1618	-151.18907±10.1714	-196.61977±0.1360	-196.6211±0.1360	m
ΔY	33.36035±0.1618	31.59312±16.9151	33.36126±0.1360	33.36129±0.1360	m
ΔZ	322.51867±0.1618	327.17669±16.8742	322.34385±0.1360	322.34378±0.1360	m
Δa	-162.99899	-	-	-	m
Δf	-255677.13631	-	-	-	
R_x	-	0.44514±1.6E-06	0.44411±1.5848E-06	0.44514±1.6042E-06	seconds

Figure 2: Control points within the Golden Triangle

Finally, statistical analysis was carried out on all parameters determined by each of the six models. A summary of the transformation procedures is shown in the flow chart (figure 3) below:

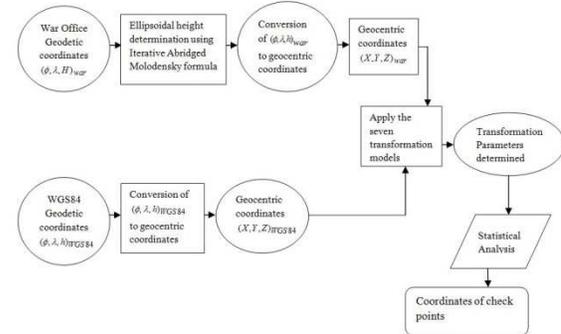


Figure 3: Computational Procedures for transformation Parameters determination

Results and Discussion

Transformation models applied

The conformal transformation models computed parameters and their corresponding statistic values for transforming War Office data to WGS84 system are shown in Table 1 below. It is evident from table 1 that the two origins of the reference ellipsoids have a negative displacement on the X-axes and positive displacement on the Y and Z-axes. The X-axes displacement direction suggests that the movements of both reference systems are in the opposite direction whilst the reverse situation exists for the Y and Z-axes. The translation parameter ΔX revealing the existence of a negative displacement from the geocenter is because both X-axes in the two reference ellipsoids are moving in opposite directions. Conversely, ΔY and ΔZ translation parameters from the geocenter evident from table 1 below show that the axes of both reference ellipsoids move in the same direction.

R_Y	-	-0.00582±2.65E-6	-0.03304±2.6477E-06	-0.00582±2.6521E-06	seconds
R_Z	-	0.02199±2.64E-6	-0.00535±2.6548E-06	0.02199±2.6397E-06	seconds
ΔS	-	-7.16775±1.58E-6	-7.16775±1.5827E-06	-7.16775±	ppm
Reference SD	0.70518	0.59297	0.59298	0.59298	m
Reference AV	0.49728	0.35162	0.35163	0.35163	m

The reference SD and reference AV from table 1 represent the reference standard deviation and reference adjustment variance. The Δa and Δf ellipsoidal change provided the scale change in conjunction with ΔX , ΔY and ΔZ in the case of the abridged Molodensky model. A Visual observation of table 1 presents a negative scale factor values for Bursa-Wolf, Veis and Molodensky-Badekas models. This negative scale factor denotes that the geometric shapes of the Ghana War Office and WGS84 system are reduced when the scale factor is applied in conjunction with the derived parameters for transforming coordinates from War Office to WGS84 datum and vice versa. Therefore, this corroborates the facts that, similarity transformation preserves shapes and angles but the lengths of lines and the position of points may change. The introduction of rotation angles in the Bursa Wolf, Veis and Molodensky models brings about massive changes in the geocentric (translation) parameters compared to the abridged Molodensky model (table 4.1). Intuitively, it is easy to see that a small rotation around the geodetic X or Y-axis leads to a substantial change in the relative position of the ellipsoid centers which is equivalent to a change in the geocentric translations shown clearly in table 1 above. The introduction of the centroid coordinate for the War Office datum in the Veis and Molodensky-Badekas model tends to eliminate the correlation of transformation parameters that exists in Bursa-Wolf model. The centroid coordinates in (X, Y, Z) used in the derivation of the parameters are; $X = 6339126.39570231$ m, $Y = -133380.293067743$ m and $Z = 689482.733775943$ m. To have an indication on the precision of the transformation to know how well the transformed coordinates from War Office system agree with the known coordinates in the WGS84 system, the reference standard deviation for the entire observation and the individual standard deviations for the computed parameters were estimated. Identical standard deviation values for the geocentric translation parameters were achieved for abridged Molodensky, Veis and Molodensky-Badekas model (table 1 above). The general results in table 1 above

show relatively smaller standard deviation values for each model parameters determined, thus, warrant its acceptance. The statistical results also indicate that there is 68% probability that the observation and its associated parameters lie between plus or minus the reference standard deviation from their estimated values in table 1 above. In addition, this standard deviation obtained will indicate a steep bell-shape on the normal distribution curve.

The position dependent methods (3D projective and 12 parameter affine) experimented in this study showed some limitations. Firstly, because there was no change in datum and the coordinate data were converted directly from the War Office to the WGS84 system. It was also observed that although the reference standard deviation for the entire data gave a better precision, the opposite was realized especially in the translation parameters when their standard deviations were estimated. This was also attributed to the fact that the distortions existing in the War Office system surveyed by conventional techniques could not be modeled and absorbed completely by the position dependent methods. It can therefore, be inferred that the position dependent methods experimented in this study can be very reliable for a smaller region (study area) in Ghana having data in the same reference coordinate system and not for national datum transformations even though the final results obtained after applying the parameters were reliably accurate. However, if the position dependent methods experimented in this study are to be applied nationally in Ghana, the author is proposing that an introduction of rotation parameters and scale factor should be introduced into the position dependent equations, thus developing a new model. This new developed model will absorb the systematic distortions in the old datum hence, improvement in its reliability and accuracy. Tables 2 and 3 present the results of the position dependent methods in meters.

Table 2: Results from 3-D Projective Transformation

3D Projective	Parameter	a1	a2	a3	a4
	Estimated value	0.630343	0.00458	-0.01956	1178583.4
	Standard Deviation	0.0562063	0.0005435	0.00292	179117.86
	Parameter	b1	b2	b3	b4
	Estimated value	0.0033657	0.8139463	0.0003518	-21614.112
	Standard Deviation	0.0008232	0.0282877	8.258E-05	5276.1054
	Parameter	c1	c2	c3	c4
	Estimated value	-0.0207096	0.000512	0.8118304	132593.31
	Standard Deviation	0.0008232	0.0282877	8.258E-05	5276.1054
	Parameter	d1	d2	d3	
	Estimated value	-2.899E-08	7.23E-10	-3.087E-09	
	Standard Deviation	4.409E-09	8.574E-11	4.611E-10	

Table 3: Results from 12 Parameter Linear Affine Transformation

12 Parameter	Parameter	a1	a2	a3	a4
	Estimated value	0.9998539	3.346E-06	-1.444E-05	-0.0002641
	Standard Deviation	0.0002469	5.631E-06	2.621E-05	0.0002469
	Parameter	a5	a6	a7	a8
	Estimated value	0.9999976	-2.799E-05	0.0012028	-3.033E-05
	Standard Deviation	5.631E-06	2.621E-05	0.0002469	5.631E-06
	Parameter	a9	X ₀	Y ₀	Z ₀
	Estimated value	1.0001182	739.98285	1726.7374	-7387.6435
	Standard Deviation	2.621E-05	1584.1411	1584.1411	1584.1411

The reference standard deviations obtained for 3D projective and 12-parameter affine models are 0.754328 m and 0.474532 m. These values show that the precision of the data set is within the range of ± 0.754328 and ± 0.474532 , respectively.

Residual Analysis from Derivation of the Parameters

The evaluation of the performance of the datum transformation methods was focused on the residuals generated in estimating the parameters. Figures 4, 5, 6, 7, 8 and 9 below, show the variation of residuals with respect to the observations (control points) when the transformation models were executed in determining their related transformation parameters. In addition, the graphs also show how much the X, Y and Z coordinates fluctuate along the ideal threshold value of zero on the vertical axes. This threshold value also known as the 90-degree line gives a better indication of the inconsistencies in the models by way of errors in the coordinates. The V_X, V_Y and V_Z on the graphs below represent residuals in the (X, Y, Z) coordinates for both systems.

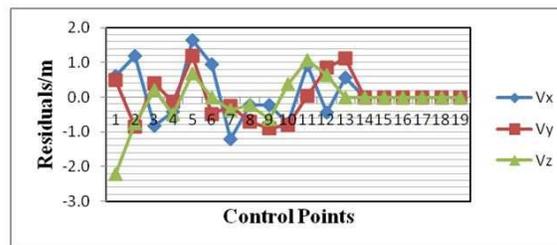


Figure 4: A graph of residuals against control points for Abridged Molodensky Model

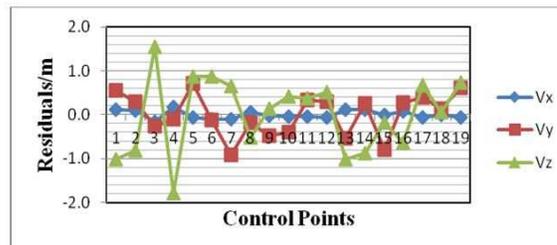


Figure 5: A graph of residuals against control points for Bursa-Wolf Model

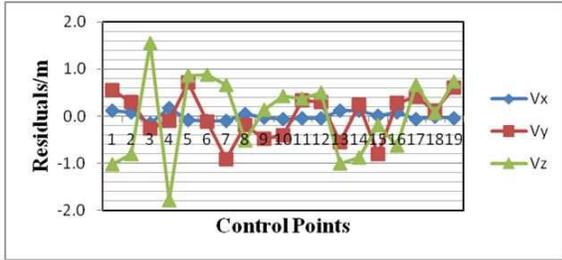


Figure 6: A graph of residuals against control points for Veis Model

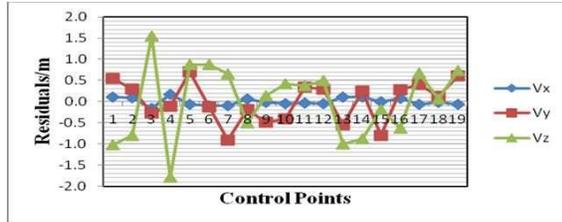


Figure 7: A graph of residuals against control points for Molodensky-Badekas Model

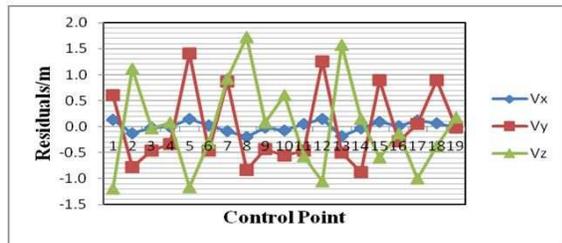


Figure 8: A graph of residuals against control points for 3D Projective Model

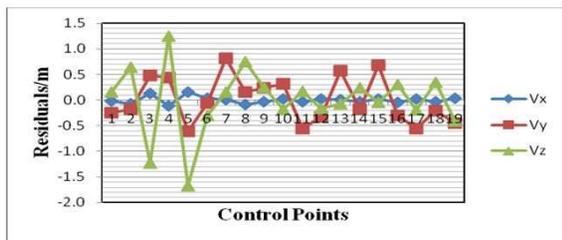
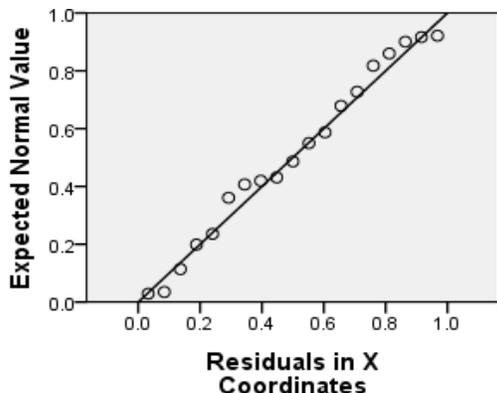


Figure 9: A graph of residuals against control points for 12-Parameter Affine Model

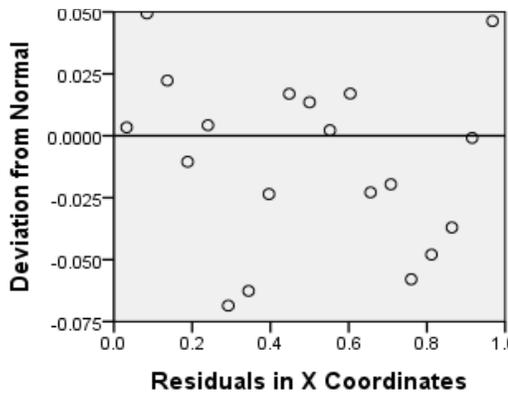
A striking observation in figures 4, 5, 6, 7, 8 and 9 above show a large V_x in the abridged Molodensky five-parameter model compared to the other transformation models where there is a graphical evidence of improvement in the horizontal fluctuation along the threshold. However, uniformity in displacement was observed upon inspection at control points GCS 302 to CFP 207 in the abridged Molodensky compared to the other transformation models.

Comparatively, the residuals in the horizontal positions of the seven-parameter similarity approach show a fairly consistent rise and fall around the threshold. Nonetheless, based on visual observations of the seven parameter similarity transformation graphs of figures 5, 6 and 7, it was seen that, in the V_z component, there was a significant rise from control point CFP 200 to CFP 225 and a downward fall from control point CFP 225 to GCS 102 and another sharp increase from control point GCS 102 to CFP 155 respectively. In the case of the position dependent, the V_x residuals are relatively smaller and closer to the displacement threshold value than the V_y residuals which have a steady rise and fall. However, the V_z component residuals in the position dependent methods had larger fluctuations around the threshold compared to all the conformal similarity models applied in this study. These inconsistencies incurred as explained above during the parameter determination can be attributed to three factors. The first is due to the distortions in scale and orientation existing in the old datum (War Office) which could not be absorbed and modeled completely by the transformation models. Secondly, the possible non-parallelism existing between the axes of the coordinate systems of WGS84 and War Office datum. Finally, random errors existing in both observation data applied in the parameter determination have an influence on the outcome of the estimation even though this depends on the confidence interval. These factors however, have contributed to the inability of the similarity transformation models to notice its potential of providing higher (sub-meter or even sub-centimeter) accuracy even though they are rigorous models.

The observational residuals obtained from each transformation model can be statistically tested and the better method can be chosen. It is therefore imperative to determine whether the residuals are normally distributed before any statistical test can be carried out. However, if the residuals are not normally distributed then the results obtained may be biased by systematic errors. Therefore, to test the normality assumptions of the residuals, normality probability plots for all the seven models were carried out. A sample of the normality probability plots of the residuals is shown in figure 10 below.



(a) Normality P-P Plots



(b) Detrended Normal P-P Plot

Figure 10: 3D Projective Normality Probability Plots for the X-Coordinate

Table 4: Residual Estimation Accuracy Measures

Transformation	Error Term	Max(-)/m	Max(+)/m	Mean Error (m)	Standard Error (m)
Abridged Molodensky	V_x	-1.200652694485370	1.632192018604710	0.090493410	0.713109842
	V_y	-0.902514314280317	1.183510564459880	-0.005235780	0.494964469
	V_z	-0.367083085805358	1.056066321658440	-0.093677902	0.624947662
Bursa-Wolf Model	V_x	-0.161517126175141	0.169559252613851	-0.000000000000012	0.08899041
	V_y	-0.918136195970270	0.709633269562069	0.000000000000000	0.464964103
	V_z	-1.795210838887440	1.54259277792660	0.000000000000075	0.837377676
Veis Model	V_x	-0.152126695960760	0.173266176134348	0.00000001666580	0.089595057
	V_y	-0.918067855876871	0.709828763647238	-0.00000000058208	0.464913725
	V_z	-1.794841314316730	1.543590024812150	0.00000000165432	0.837359175
Molodensky-Badekas Model	V_x	-0.161517125554383	0.169559250585735	0.00000001666580	0.088990409
	V_y	-0.918136195716215	0.709633269143524	-0.00000000059739	0.464964103
	V_z	-1.795210837968620	1.542592778103420	0.00000000214450	0.837377676
3D Polynomial Projective Model	V_x	-0.199499413672494	0.148815443237087	0.000000000000000	0.102491526
	V_y	-0.872173108872741	1.408923863644180	0.000000000000000	0.72155547
	V_z	-1.188935449226870	1.727083708193740	0.000000000000000	0.852446501
12 Parameter Affine Model	V_x	-0.108525931835175	0.159129785373807	0.000000000686238	0.062490749
	V_y	-0.611474014169289	0.809538537432673	0.00000000031880	0.435115889
	V_z	-1.668185951071790	1.247234170441520	0.00000000042890	0.631946059

For the statistical tests of the residuals (V_x , V_y , V_z), the maximum negative error, maximum positive error, mean error and standard error values were calculated as shown in table 4 above. The analysis of the maximum negative error, maximum

positive error and mean error values given in table 4 shows that, the seven-parameter similarity transformation models are more than three times better than the five parameter Molodensky model in both X, Y and Z respectively. However, in the case

of the position dependent methods it was realized that the 3D projective model gave negligible values of mean errors compared to the 12 parameter affine model. In addition, it can be seen from table 4 above that the horizontal coordinates (X, Y) are much better than the vertical coordinate (Z). This arises from the different horizontal and vertical surveys of the Ghana War Office datum. A careful study of table 4 above shows that the seven-parameter models estimated the 3D coordinates (X, Y, Z) with a significantly better accuracy than the abridged Molodensky in terms of the standard error. Comparatively, both the similarity based datum shift and position dependent methods gave closer accuracies

Checking the Statistical Validity of the Derived Transformation Parameters

A hypotheses test for each of the transformation parameters derived in the computational process was performed at 5% level of significance. This was done to ascertain whether all the values of the parameters determined could be

judged statistically different from zero and thus, significant. Tables 5 and 6 below presents the calculated **t** statistic results for both similarity based datum shift and position dependent transformation methods. In order to test the hypothesis, the measures of dispersion (standard deviation) around the ‘true’ mean were calculated.

Testing of Hypothesis: $H_0: \beta_1 = 0$ (Each parameter is not statistically different from zero)

$H_1: \beta_1 \neq 0$ (Each parameter is statistically different from zero)

Significance Level: $\alpha = 0.05$

Test Statistic: $t = \frac{|Parameter|}{SD}$ where SD is the

standard deviation of the individual parameters shown in table 1.

Decision Rule: Reject H_0 if $t > t_{(\alpha/2, v)}$, where V is the degree of freedom.

Conclusion: If the calculated t is greater than $t_{\alpha/2, v}$, reject the null hypothesis of that parameter and vice versa.

Table 5: Computed t-test Values for Similarity Based Datum Shift Methods

Parameter/Transformation	t_x	t_y	t_z	R_x	R_y	R_z	S
Abridged Molodensky	1215.40	206.2090	1993.570	-	-	-	-
Bursa-Wolf	14.86418	1.86775	19.38915	277471.77025	2193.15324	8330.58422	4.52883
Veis Model	1445.27034	245.22024	2369.37023	280236.69171	12479.83665	2013.44821	4.52875
Molodensky-Badukas	1445.34312	245.23572	2369.51835	277471.77046	2193.15214	8330.58223	4.52883

Table 6: Computed t-test Values for Position Dependent Methods

3D Projective	Parameter	a1	a2	a3	a4	b1	b2	b3	b4
	t-value	11.21482	8.43106	6.69552	6.57993	4.0887	28.7738	4.25996	4.0966
	Parameter	c1	c2	c3	c4	d1	d2	d3	
	t-value	25.15847	0.0181	9830.7	25.1309	6.57449	8.43268	6.6952	
12 Parameter	Parameter	a1	a2	a3	a4	a5	a6	a7	a8
	t-value	4048.84493	0.59417	0.55087	1.06961	177586.88429	1.06777	4.87050	5.38632
	Parameter	a9	X_0	Y_0	Z_0				
	t-value	38159.37390	0.46712	1.09001	4.66350				

From student t-distribution table, the following critical values (table 7) were obtained.

Table 7: Tabular t values

Transformation Models	$t_{(\alpha/2, v)}$
Abridged Molodensky	2.145
7-Paramater Similarity	2.179
3D Projective	2.776
12 Parameter	2.365

Judging from the t values obtained (tables 5 and 7) for the abridged Molodensky model parameters, it can be concluded that all parameters are significantly different from zero at 95% confidence interval, hence, $H_0:\beta_1 = 0$ is rejected. Also, by comparing the tabular t value (table 5) against its related computed value (table 7) in the case of the seven parameter similarity models, all geocentric translation values, rotations and their respective scale factors were significantly different from zero at 95% confidence interval. Hence, the null hypothesis ($H_0:\beta_1 = 0$) for these parameters were rejected. Therefore, there is a non-zero relationship between the War Office and WGS84 reference coordinate system. As a result, the conclusion to be drawn here is that, the parameters of the abridged Molodensky model and the seven-parameter similarity models applied in this study are all statistically significant. However, the position dependent methods proved otherwise. It is evident by making comparison between table 6 and 7 that, not all parameters calculated are statistically different from zero. This is because coordinate positions are directly transformed between the reference systems when the parameters were derived hence, having a major influence on the standard deviations of the parameters determined. This further corroborates that the position dependent methods do have a close association between the War Office and WGS84 reference coordinate system, thus, not all parameters are statistically significant. Nevertheless, the position dependent methods can still be applied for local survey work, for example, in Ghana where coordinates are in the same system. This will tend to some extent reduce the discrepancies existing when it is applied for parameter determination between two different datums, for example, WGS84 and Ghana War Office.

Comparison of Bursa-Wolf, Molodensky-Badekas and Veis Models

There are several significant differences in the Bursa-Wolf, Molodensky-Badekas and Veis Models. Based on the knowledge and properties of the models, comparisons were made on their respective similarities and differences in determining the transformation parameters. All three models contain a maximum of seven unknown transformation parameters obtained via a parametric least squares estimation procedure.

From the Bursa-Wolf model equation (Deakin, 2006) and the Molodensky-Badekas model equation (Deakin, 2006), the two transformations can be written in vector form as

$$I_2 = t_B + (1 + \Delta S_B)R_B I_1 \tag{3}$$

$$I_2 = t_M + (1 + \Delta S_M)R_M \bar{I}_1 \tag{4}$$

Where:

t_M and t_B are Bursa-Wolf and Molodensky-Badekas translations .

$\Delta S_B, \Delta S_M$ are Bursa-Wolf and Molodensky-Badekas models scale factors.

R_B and R_M are Bursa-Wolf and Molodensky-Badekas models rotations respectively.

Constructing similarity transformation equations in terms of the coordinate differences computed from the origin to the i th point, equation (3) and (4) can be rewritten as:

$$\Delta X_{io(B)} = (1 + \Delta S_B)R_B (R_X, R_Y, R_Z)(X_i - X_0) \tag{5}$$

$$\Delta X_{io(M)} = (1 + \Delta S_M)R_M (R_X, R_Y, R_Z)(X_i - X_0) \tag{6}$$

It is obvious from equations (5) and (6) that both models are identical due to the cancellations of the translation parameters in equations (3) and (4) respectively. Since ΔI must be the same from both equations; and I_{io} is the same, the equality of equations (5) and (6) implies the following:

$$\Delta S_B = \Delta S_M \tag{7}$$

$$R_B (R_X, R_Y, R_Z) = R_M (R_X, R_Y, R_Z)$$

This means from equation (7) that, the scale factor and the rotational parameters derived from executing Bursa-Wolf and Molodensky-Badekas models should yield identical results. This mathematical analogy (equation 7) presented was further substantiated with reference to table 1 (Bursa-Wolf and Molodensky-Badekas results) where identical values were obtained for the scale factor and rotational parameters of the two models respectively. However, the translation components obtained after the models were executed differ from each other. The reason for this is that, with reference to Bursa-Wolf model equation (Deakin, 2006) it can be seen that the scale factor S is applied to all position vectors, including the terrestrial initial point while in the case of Molodensky-Badekas model the scale difference parameter S , is applied only to the coordinate differences. It is also clear from Bursa-Wolf and Molodensky-Badekas equations that, the translation component of the Molodensky-Badekas model equals that of the Bursa-Wolf model only when the centroid coordinates is equal to zero or the

scale factor is equal to one and the rotation is an identity matrix.

Therefore equating equations (3) and (4) gives

$$(1 + \Delta S)R_s \bar{I}_1 + t_M = (1 + \Delta S)R_s I_1 + t_B \quad (8)$$

Let $S = 1 + \Delta S$ and $\bar{I}_1 = I_1 - g_1$ as defined in Deakin, 2006. Hence, substituting and expanding equation (8) becomes

$$SR_s I_1 + t_B = SR_s I_1 - SR_s g_1 + t_M$$

$$t_B = t_M - SR_s g_1 \quad (9)$$

The resulting equation shows the mathematical relationship between the Bursa-Wolf translations and Molodensky-Badekas Translations. Moreover, equation (9) depicts the outcome of the rotations and the scale factor on the translation in the Bursa-Wolf model. The magnitude of this effect is the $-SR_s g_1$. Several investigators such as (Thomson, 1974; Kutoglu et al., 2002; Deakin, 2006) have carried out this translation test. A fundamental difference between Molodensky-Badekas, Veis and Bursa-Wolf models is that, the latter has no provision for an initial point of the second network thereby treating all network points equivalently.

Recalling equations (3) (Bursa-Wolf model) and Veis model equation (Rapp, 1993), the two models can be represented in vector form as

$$I_2 = t_B + (1 + \Delta S_B)R_B I_1 \quad (10)$$

$$I_2 = (1 + \Delta S_V)R_V \bar{I}_1 + t_V \quad (11)$$

Where:

t_V is the Veis model translation

ΔS_V is the Veis model scale factor

R_V is the Veis model rotation

Comparing the new coordinates difference computed with Bursa-Wolf system with those computed by the Veis system from the origin to the i th point, equations (10) and (11) in the new system can be written as:

$$\Delta X_{io(B)} = (1 + \Delta S_B)R_B (R_X, R_Y, R_Z)(X_i - X_0) \quad (12)$$

$$\Delta X_{io(V)} = (1 + \Delta S_V)R_V (\phi_0, \lambda_0, \alpha, \xi, \eta)(X_i - X_0) \quad (13)$$

Because ΔX and $(X_i - X_0)$ must be the same from both equations, equating equations (12) and (13) implies the following:

$$\Delta S_B = \Delta S_V$$

$$R_B (R_X, R_Y, R_Z) = R_M (\phi_0, \lambda_0, \alpha, \xi, \eta) \quad (14)$$

It is obvious from equation (14) that the scale factor in the Veis model and Bursa Wolf system must be the same after parameter estimation. This mathematical theorem as stated in equation (14) was further corroborated with the results obtained in this study reference to table 1 where the same scale factor of -7.16775 was obtained for both Veis and Bursa Wolf models respectively. However, the rotation parameters did not yield identical results although the Veis model is mathematically comparable to Molodensky-Badekas model. This is because of the orientation of the Veis model in the coordinate system at the initial point. To estimate the relationship between the two rotation parameters, the Veis model equation must be equated to Molodensky-Badekas equation. In addition, the translation components evident from table 1 differ from each other. The same analogy as discussed in the case of Bursa-Wolf and Molodensky-Badekas models translations explains the situation. The Molodensky-Badekas and Veis models are mathematically equal to each other. Both models require an initial point in the terrestrial network, apply their respective rotations and scale differences to model the errors in the terrestrial network. Furthermore, these models are based on the same a priori assumption of parallelism of coordinate system axes. In addition, both models use the same observables in their respective equations. The only fundamental difference existing between Molodensky-Badekas and Veis models is the rotations at the initial point of the coordinate system. However, based on the results tabulated in 1, it was noticed that this fact does not have any bearing on the final solution or interpretation of the final transformation results. That is, identical results were achieved for the translation components and scale factors with small changes in the rotations that do not have any major influence in the final coordinates transformed. It is also worth mentioning that unless the localized datum shift (centroid) are used, the Molodensky-Badekas and Veis models do not give a best account for variable scale, survey errors and distortions within the War Office local geodetic network, thus, requiring more terms other than the other similarity transformation methods due to non homogeneity in local geodetic datums .

Comparison between Position Dependent Methods

There are a number of significant differences and similarities in the 3D Polynomial Projective and 12 Parameter Linear Affine models based on their properties and applications in

determining their transformation coefficients via a parametric least squares estimation procedure. Comparison of both models describes a change in the two coordinate system and do not include a change of datum. That is, positions are directly calculated into the other coordinate systems. In the 12 parameter affine transformation, there is an allowance of rotation, translation, skew and differential scaling on the geometric network while preserving parallelism of lines. This method maps point in one coordinate reference system into their geometrical points in another coordinate reference system. In addition, there is also an assumption that coordinates of War Office systems has a linear relationship of coordinates of the WGS84 system. On the other hand, the projective transformation allows both angular and length distortions, thus, allowing the geodetic network coordinates to be transformed into an irregular shape. The major advantage of the affine transformation over the projective is its linearity equations, thus, making it easy to use for the transformation parameter determination.

In the case of the projective transformation, the equations require to be linearized and initial approximate values closer to the 'true' parameter values are needed before estimating the parameters. This iterative nature of the model makes estimation of the parameters cumbersome. Based upon inspection of the 3D Projective equations (Ghosh, 1979; Burtch, 2010), it was noticed that, when their respective denominator parameters are equal to zero a 12 Parameter linear affine model is formed. Finally, based on the results obtained, the 12parameter affine transformation model was able to estimate the positions of coordinates to a better accuracy than the 3D projective transformation method.

Test Results Analysis

In order to compare the validity and the stability of the solutions obtained from both transformation techniques; nineteen check points (existing coordinates) were utilized. A summary of the coordinate differences between the projected and existing coordinates are shown in table 8 below.

Table 8: Summary of coordinate differences test results

Transformation	Maximum E (m)	Minimum E (m)	Maximum N (m)	Minimum N (m)
Abridged Molodensky	0.90074	-1.12094	1.87158	-2.07593
Bursa-Wolf	0.91941	-0.70927	1.80296	-1.55013
Veis	0.91930	-0.70943	1.80293	-1.55023
Molodensky-Badekas	0.91941	-0.70930	1.80296	-1.55010
3D Polynomial Projective	1.40947	-0.87246	1.73641	-1.19508
12 Parameter Affine	0.80919	-0.60970	1.25236	-1.67940

From table 8 above, it can be seen that the overall accuracy of abridged Molodensky and 3D polynomial projective techniques are in the order of approximately 0.9 meter and 1.4 meters in the Eastings and 1.9m and 1.7m in the Northings respectively. This implies that the abridged Molodensky model horizontal accuracy achieved in this study was higher than the anticipated standard accuracy level of $\pm 5m$. This improvement in accuracy means that, the distortions in the War Office datum are at a minimum. A careful study of the results obtained for the 7-parameter similarity transformation models reveal that, the CFP 180R control points recorded 0.9 meter in the Eastings and GSC 102 had 1.80 meters coordinate differences in the Northings. Conversely, the rest of the controls were within 0.6 meter and 1.0 meter coordinate

differences for Eastings and Northings which meet the standard accuracy requirement for 3-dimensional similarities of 1m. The inference to be drawn here based on the above mentioned evidence from the results is that, the control points CFP 180R and GSC 102 are acting as an outlier and should be resurveyed and checked for stability by the Ghana Survey Department. Furthermore, the 12-parameter affine transformation buttress this inference made above because a similar situation was also observed for the controls mentioned above. The computed differences between the projected coordinates of the test points represented graphically in figures 11 and 12 clearly reflect the residuals for the nineteen coordinates test points.

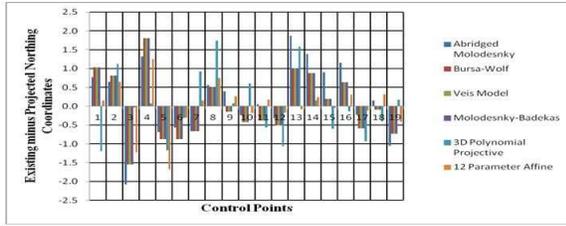


Figure 11: Northing Coordinate Differences

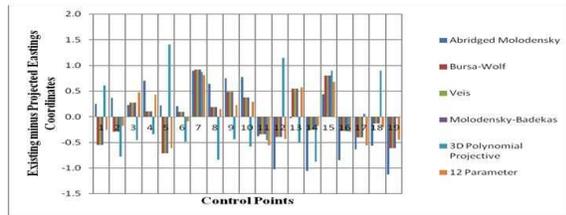


Figure 12: Easting Coordinate Differences

To evaluate the performance and adequacy of the datum transformation techniques applied in this study, the coefficient of determination, correlation coefficient and root mean square error were the statistical estimation accuracy measures utilized. These estimation accuracy measures help in assessing the values of the estimation models as well as displaying them graphically. The statistical values (R^2 and r) and the root mean square error (RMSE) values of the test data set's coordinate residuals are presented in table 9 below.

Table 9: Results from statistical test

Transformation		R^2	r	RMSE (m)
Abridged Molodensky	Eastings	1.0	1.0	0.68389
	Northings	1.0	1.0	0.99162
Bursa-Wolf Model	Eastings	1.0	1.0	0.48042
	Northings	1.0	1.0	0.86180
Veis Model	Eastings	1.0	1.0	0.48043
	Northings	1.0	1.0	0.86183
Molodensky-Badekas Model	Eastings	1.0	1.0	0.48042
	Northings	1.0	1.0	0.86180
3D Polynomial Projective Model	Eastings	1.0	1.0	0.73460
	Northings	1.0	1.0	0.87652
12 Parameter Affine Model	Eastings	1.0	1.0	0.45089
	Northings	1.0	1.0	0.65101

In this study, the coefficient of determination (R^2) was used as a criterion to measure the adequacy of the transformation techniques predictions (projected coordinates) when their related transformation parameters were applied to the observations. On the basis of the results (Table 9), the R^2 values for all the transformation techniques yielded 1 respectively. These values of R^2 indicated that the models describe the variation in the data with highly reliable accuracy. The coefficient of correlation (r) corroborated this high strength of linear dependence between the existing and projected coordinates. A sample of the correlation graphs are shown in figure 12 below.

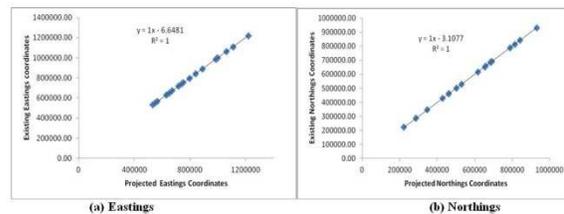


Figure 12: Scatter plots of existing against projected coordinates for Abridged Molodensky

To further confirm the degree of linearity (correlation) between actual and projected coordinates for the transformation techniques, a test of hypotheses (t statistic) at 5% significance level was conducted on the correlation coefficients determined.

Testing of Hypothesis:

Null hypothesis: Existing coordinates do not have any correlation with the projected coordinates

$$H_0 : \rho = 0$$

Alternative Hypothesis: Existing coordinates do have correlation with the projected coordinates

$$H_1 : \rho \neq 0$$

Significance Level: $\alpha = 0.05$

Test Statistic:
$$t = r \frac{\sqrt{n-2}}{\sqrt{1-r^2}}$$

Where; r = correlation coefficient (refer to table 9)
n = number of observations = 19

Decision Rule: Reject H_0 if $|t| > t_{\alpha/2, n-2}$

Conclusion: If the calculated $|t|$ is greater than $t_{\alpha/2, n-2}$, reject the null hypothesis and vice versa.

From the student t-distribution tables, $t_{(0.025, 17)} = 2.110$. Observation of the test statistic equation above shows that the test statistic is at infinity and therefore the test will be at the tail end of the distribution; this makes it highly significant. Hence, the null hypothesis is rejected and the alternative accepted since the calculated t is greater than the critical value. This further confirms that the results established a strong correlation between the existing and projected coordinates.

The RMSE shown in table 9 above, were used in the evaluation process of the transformation methods because the RMSEs are sensitive to even small errors and can determine the quality of a transformation, making it good in comparing small changes between projected and observed differences in models. From table 9 above, it is evident that the 12 parameter affine model estimated the 2D coordinates (E, N) of the test points with a significantly better accuracy than the other transformation models in terms of the RMSE. That is, the 12 parameter affine gave the smallest RMSE values (0.45089 m, 0.65101 m) of the 2D coordinate residuals. In addition, the 7-parameter similarity transformation methods produced identical results of RMSE values. This further confirms the assertion that there is no much significant difference between the 7-parameter similarity transformation models.

Figure 9 below, is a root mean squared error distribution in both Eastings and Northings for the transformation models. This distribution shows the accuracy of the transformation models estimations for a set of test data and determines how well the estimation performed. The bar chart shows the root mean square error distribution and the bar height represents the root mean square of the difference between the existing coordinates that fall within the range of the bin and their projected coordinate

values. Moreover, to ensure the accuracy of the transformation, the RMSE should be within a tolerance value. If the RMSE is within the acceptable range, then it is assumed that the transformation of the entire coordinates is acceptable. By comparing the RMSE's values to the achievable accuracy standard for the models, it can fairly be stated that the transformation is within tolerance.

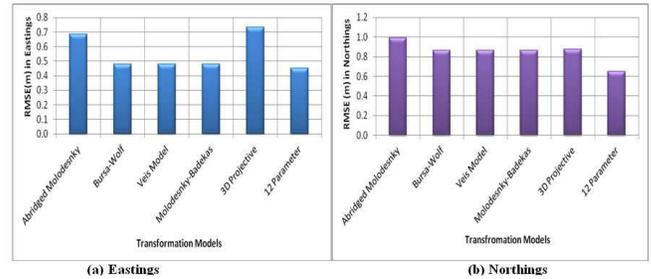


Figure 4.12: A graph of RMSE in Eastings and Northings for the transformation models

Visual observation of figure 9 shows that the estimations in Northings coordinates gave a lower accuracy compared to the Eastings. This may perhaps be attributed to the effect of the height determined for the Ghana War Office ellipsoid when the Orthometric height were converted into ellipsoidal height using iterative abridged Molodensky since there is no existence of geoid in Ghana. Thus, the geoid undulation could not be determined. Therefore, applying the Z coordinate in the inverse computation of the latitude will incur some errors that will affect the final projected coordinates in Northings. However, the projected Eastings coordinates of the Ghana War Office system were better because the computation of the longitude uses the horizontal coordinates (X, Y) which are determined to a better accuracy.

Conclusion

Coordinate transformations are necessary to convert War Office coordinates to the GPS based WGS84 coordinates system. In order to obtain an accurate and consistent transformation parameter sets in Ghana, it requires a herculean task due to the inaccuracies and inconsistencies in the War Office geodetic network. In this study, datum transformations between War Office and WGS84 involving two-dimensional (2D) and three-dimensional (3D) coordinates systems have been studied. This procedure involves combining axes rotations, scale change and origin shift by applying transformation equations. A 3D similarity transformation comprising of Bursa-Wolf, Veis and Molodensky-Badekas models have been investigated. In addition, position dependent methods

namely; 3D polynomial projective and 12 parameter linear affine models were also studied. The abridged Molodensky transformation model was also investigated within the Golden Triangle. Based on the results, it was found that both similarity and position dependent methods produce results consistent with their accuracies that have been quoted in scientific literature. It was also found that the 3D similarity methods do not exhibit any significant differences based on their identical results and precision estimates obtained. Transformation of WGS84 coordinates to the War Office coordinates system can be expected to be within $\pm 1\text{m}$ of the War Office values without any outliers for the 3D similarity models. The results further confirm the uncertainty of 1m existing in 3D similarity methods. The obtained results are a solution for majority of GIS and mapping problems as well as for cadastral work in Ghana. Moreover, the comparison of these 3D similarity models revealed that the Veis and Molodensky-Badekas models were superior to Bursa-wolf model. Hence, it is concluded in this study that the Veis and Molodensky-Badekas models are the most precise 3D similarity transformation model within the Golden Triangle. It must be noted that, before the transformation parameters from the Veis and Molodensky-Badekas model are to be applied to transform coordinates of points within the Golden Triangle, it is essential for the user to know the centroidal coordinate system values used in the derivation of the parameters. This will enable the user to know the positions about which the coordinate reference frame was rotated in the War Office local geodetic network. On the other hand, the abridged Molodensky values based on the results at the test points can produce War Office coordinates accuracy to $\pm 1.9\text{m}$ better than the anticipated accuracy of $\pm 5\text{m}$. The results suggest that this model can be suitable, for example, for farm compensation surveys, reconnaissance survey using hand-held GPS where higher accuracies are not required. In the case of the position dependent methods, the 12-parameter linear affine model gave better estimation accuracy within $\pm 0.8\text{m}$ compared to the 3D projective model. Numerical investigations revealed that the 12-parameter affine model is superior to the seven-parameter similarity models. This corroborates the scientific findings that the higher the number of parameters in a model; existing in the 12-parameter affine, the better the model fit and a reduction in residuals. In spite of this, the 12-parameter affine model cannot be chosen as the preferred method for transformation of data between the two coordinate reference systems within the Golden Triangle. This is

because, not all of the 12-parameter affine model parameters were statistically significant when the t-test statistic was performed. Therefore, the 3D similarity transformation models that satisfied all the numerical tests are recommended as the proposed methods for datum transformation within the Golden Triangle. However, based on the author's opinion, the 3D similarity method is also not the optimal transformation model for the Golden Triangle. This is because the criteria for choosing the most optimal transformation model are beyond the scope of this work. Therefore, this study can only propose a better method based on empirical evidence from the numerical investigations but cannot state categorically the optimal model. This is in line with the objectives of this study. A conclusion was drawn from the numerical results that the accuracies of the derived transformation parameters are dependent on the spatial distribution of the geodetic network than on the number of observation points. It was also concluded from the results that, the main cause of obtaining different transformation parameter estimates by researchers in Ghana for the Golden Triangle was attributed to the War Office geodetic network formed by conventional procedures. These networks have, for many reasons, distortions most likely originating from systematic and random errors found in the coordinates of the points. These errors are largely produced by low accuracy heights obtained mostly by trigonometric leveling in the War Office networks and by the absence of geoid undulations required for the conversion to ellipsoidal heights. To conclude, issues of datum transformation of GPS coordinates from the War Office to the WGS84 coordinate systems are a real, persistent and serious problem in Ghana that requires pragmatic measures for solutions.

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